

06MAT31

Third Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - III
Time: 3 hrs .
Max. Marks:100

## Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. Expand the function $\mathrm{f}(\mathrm{x})=\mathrm{x}-\mathrm{x}^{2}$ in the interval - $\pi<\mathrm{x}<\pi$. Deduce that $\frac{\pi^{2}}{12}=\frac{1}{1^{2}}-\frac{1}{2^{2}}+\frac{1}{3^{2}}-\frac{1}{4^{2}} \ldots \ldots$.
(07 Marks)
b. Find the half-range cosine series for the function $f(x)=(x-1)^{2}$ in $0<x<1$.
(07 Marks)
c. The following table gives the variations of periodic current over a period

| $\mathrm{t}(\mathrm{sec}):$ | 0 | $\mathrm{~T} / 6$ | $\mathrm{~T} / 3$ | $\mathrm{~T} / 2$ | $2 \mathrm{~T} / 3$ | $5 \mathrm{~T} / 6$ | T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~A}(\mathrm{amp}):$ | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the $1^{\text {st }}$ harmonic.
(06 Marks)
2 a. Express the function $f(x)=\left\{\begin{array}{ll}1, & |x|<1 \\ 0, & |x|>1\end{array}\right.$ as a Fourier integral and hence evaluate $\int_{0}^{\infty} \frac{\operatorname{Sin} x}{x} d x$.
b. Find Fourier sine transform of $\frac{1}{x} \mathrm{e}$
(07 Marks)
c. Use convolution theorem to find the inverse Fourier transform of $\frac{1}{\left(1+\mathrm{s}^{2}\right)^{2}}$ given that $\frac{2}{1+\mathrm{s}^{2}}$ is the Fourier transform of $\mathrm{e}^{-|x|}$.
(06 Marks)
3 a. Form the parfal differential equation by eliminating the arbitrary function from $Z=y^{2}+2 f\left(\frac{1}{x}+\log y\right)$.
(07 Marks)
b. Solve $\frac{\partial^{2} z}{\partial x \partial y}=\operatorname{Sin} x \operatorname{Sin} y$, given that $\frac{\partial z}{\partial x}=-2 \operatorname{Sin} y$, when $x=0$; and $z=0$ when $y$ is an odd multiple of $\pi / 2$.
(07 Marks)
c. Solve $x\left(y^{2}-z^{2}\right) p+y\left(z^{2}-x^{2}\right) q=z\left(x^{2}-y^{2}\right)$. (06 Marks)

4 a. Derive the one dimensional heat equation in the standard form.
(07 Marks)
b. Obtain the various solutions of the Laplace's equation $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$ by the method of separation of variables.
(07 Marks)
c. A string stretched between the two fixed points $(0,0)$ and $(1,0)$ and released at rest from the position $y=\lambda \operatorname{Sin}(\pi x)$. Show that the formula for its subsequent displacement $y(x, t)$ is $\lambda \operatorname{Cos}(c \pi t) \operatorname{Sin}(\pi x)$.
(06 Marks)

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1 \text { of } 2
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## PART - B

5 a. Show that a real root of the equation $\tan x+\tan h x=0$ lies between 2 and 3 . Then apply the regula falsi method to find the third approximation.
(07 Marks)
b. Apply Gauss - Jordan method to solve the system of equations:
$2 \mathrm{x}+5 \mathrm{y}+7 \mathrm{z}=52 ; 2 \mathrm{x}+\mathrm{y}-\mathrm{z}=0 ; \mathrm{x}+\mathrm{y}+\mathrm{z}=9$.
(07 Marks)
c. Use power method to find the dominant eigen value and the corresponding eigen vector of the matrix $\mathrm{A}=\left[\begin{array}{ccc}2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2\end{array}\right]$ with the initial eigen vector as $[1,1,1]^{\mathrm{T}}$.
(06 Marks)

6 a. Under the suitable assumptions find the missing terms in the following table:

| $x:$ | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(\mathrm{x}):$ | 2.6 | - | 3.4 | 4.28 | - | 14.2 | 29 |

b. Use Newton's divided difference formula to find $f(4)$ given.
(07 Marks)

| $x:$ | 0 | 2 | 3 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x):$ | -4 | 2 | 14 | 158 |

(07 Marks)
c. Using Simpson's $3 / 8$ rule, evaluate $\int_{0}^{0.3} \sqrt{1-8 x^{3}} d x$, by taking 7 ordinates.
(06 Marks)

7 a. Solve the variational problem $\left.\delta \int^{\pi / 2}\left((y)^{2}-\left(y^{\prime}\right)\right)^{2}\right)$ dx under the conditions y $(0)=0$, y $(\pi / 2)=$ 2.
(07 Marks)
b. Find the curve on which the function $\int_{0}^{\pi / 2}\left[(y)^{2}-\left(y^{\prime}\right)^{2}-y \operatorname{Sin} x\right] d x$ under the conditions $y(0)=y(\pi / 2)=0$, can be extremised.
(07 Marks)
c. Prove that the catenaty is the plane curve which when rotated about a line ( $x-$ axis) generates a surface of revolution of minimum area.
(06 Marks)
8 a. Find he 7 transform of i) $n^{2}$; ii) $n e^{-a n}$.
(07 Marks)
b. Prove that 1) $Z(\cos n \theta)=\frac{z(z-\operatorname{Cos} \theta)}{z^{2}-2 z \operatorname{Cos} \theta+1}$; ii) $z(\operatorname{Sin} n \theta)=\frac{z \operatorname{Sin} \theta}{z^{2}-2 z \operatorname{Cos} \theta+1}$.
(07 Marks)
c. Find the inverse Z - transform of $\frac{\mathrm{Z}}{(\mathrm{Z}-1)(\mathrm{Z}-2)}$.
(06 Marks)

# Third Semester B.E. Degree Examination, May/June 2010 Materials Science and Metallurgy 

Time: 3 hrs .
Max. Marks:100

## Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART - A

1 a. Calculate the atomic packing factor of a FCC crystal lattice. (04 Marks)
b. Iron has an atomic radius of $0.124 \mathrm{~nm}, \mathrm{BCC}$ crystal structure and an atomic weight of $55.85 \mathrm{~g} / \mathrm{mol}$. Calculate its density.
(04 Marks)
c. Differentiate between edge and screw dislocations, with sketches.
(0,8 Marks)
d. State and explain Fick's first law of diffusion.
(04 Marks)

2 a. Draw the stress-strain curve for the following materials :
i) Mild steel
ii) Copper
iii) Cast iron
(03 Marks)
b. A cylindrical specimen of medium carbon steel, having an original diameter of 20 mm , when subjected to a tension test has a fracture strength of 450 MPa . If its final diameter at fracture is 12 mm , calculate the engineering stress, engineering strain and true stress.
c. Differentiate between slip and twinning.
(06 Marks)
d. Derive an expression for critical resolved shear stress for slip, with a sketch.
(06 Marks)
(05 Marks)
3 a. Explain with a sketch, how a fatigue test is carried out.
(07 Marks)
b. Differentiate between ductile and brittle fractures, with sketches.
(07 Marks)
c. Discuss any two mechanisms for creep.
(06 Marks)
4 a. Distinguish between substîtutional and interstitial solid solutions.
(04 Marks)
b. Differentiate between eutectic and peritectoid transformations, with sketches.
(06 Marks)
c. Two metals A and B having melting points of $800^{\circ} \mathrm{C}$ and $1100^{\circ} \mathrm{C}$ respectively, form an eutectic alloy at $500^{\circ} \mathrm{C}$ with an eutectic composition of $65 \%$ B and $35 \% \mathrm{~A}$. They have unlimited liquid solubilities. The solid solubilities of B in A are $12 \%$ at $500^{\circ} \mathrm{C}$ and $6 \%$ at room temperature. The solid solubilities of A in B are $10 \%$ at $500^{\circ} \mathrm{C}$ and $5 \%$ at room temperature. Draw the complete phase diagram and label all the fields. Determine the number, type, composition and relative amounts of phases present, at room temperature, for an alloy of $30 \% \mathrm{~B}$ and $70 \% \mathrm{~A}$.
(10 Marks)

## PART - B

5 a. Explain the solidification process of hypereutectoid steel with $1.2 \% \mathrm{C}$, when it is cooled from a temperature of $950^{\circ} \mathrm{C}$ to $600^{\circ} \mathrm{C}$. Draw the microstructures and the cooling curve.
(08 Marks)
b. Determine the percentages of pro-eutectoid ferrite, eutectoid ferrite and eutectoid cementite for $0.6 \% \mathrm{C}$ hypoeutectoid steel at $720^{\circ} \mathrm{C}$.
(06 Marks)
c. Draw a neat TTT diagram for eutectoid steel and indicate all the phases.
(06 Marks)
6 a. Differentiate between normalizing and annealing, with a sketch.
b. Discuss the precipitation hardening of $\mathrm{Al}-4 \mathrm{wt} \% \mathrm{Cu}$ alloy.
c. Explain induction hardening, with a sketch.

7 a. Compare grey cast irol 1 with S.G. iron, with respect to their structure, composition, properties and applic ations.
b. Explain th $\geqslant$ compositic m , properties and applications of :
i) $\mathrm{Al}-\mathrm{Si}$ a lloys ii $\mathrm{Cu}-\mathrm{Zn}$ alloys
(08 Marks)
c. Define harc enability a nd hardness.
(04 Marks)

8 a. Write short t totes on: i) Crevice corrosion ii) Stress corrosion
b. Explain with a neatsk etch, a galvanic cell.
c. Compute the voltage. it $25^{\circ} \mathrm{C}$ of an electrochemical cell consisting of pure Cd immersed in a $2 \times 10^{-3} \mathrm{M} \mathrm{sc}$ lution of $\mathrm{Cd}^{2+}$ ions and pure Fe in a 0.4 M solution of $\mathrm{Fe}^{2+}$ ions. Also write the spontaneous reaction. The half-cell potentials for Cd and Fe are -0.403 and -0.440
respectively.


# Third Semester B.E. Degree Examination, May/June 2010 

 Basic ThermodynamicsTime: 3 hrs .
Max. Marks:100

## Note: 1. Answer any FIVE full questions, selecting at least TWO questions from each part. <br> 2. Use of thermodynamics data handbook is permitted.

PART - A

1 a. What are intensive, extensive and specific extensive properties? Give examples of each.
(06 Marks)
b. What is a quasistatic process? What is its characteristic feature?
(04 Marks)
c. A thermometer is calibrated with ice and steam points as fixed pointsreferred to as $0^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$ respectively. The equation used to establish the scale is $t=\operatorname{alog}_{\mathrm{e}} \mathrm{x}=\mathrm{b}$
i) Determine the constants ' $a$ ' and ' $b$ ' in terms of ice point ( $x_{1}$ ) and stam point ( $x_{s}$ ).
ii) Show that $t^{\circ} \mathrm{C}=100 \frac{\log _{\mathrm{e}}\left(\frac{\mathrm{x}}{\mathrm{x}_{\mathrm{i}}}\right)}{\log _{\mathrm{e}}\left(\frac{\mathrm{x}_{\mathrm{S}}}{\mathrm{x}_{\mathrm{I}}}\right)}$
(08 Marks)
d. Define thermodynamic equilibrium.
(02 Marks)
2 a. Does heat transfer inevitably cause a temperature rise? What is the other cause for rise in temperature?
(05 Marks)
b. Define thermodynamic work. Briefly explain, what is displacement work.
(07 Marks)
c. A mass of gas is compressed in a quasistatic process from $80 \mathrm{Kpa} ; 0.1 \mathrm{~m}^{3}$ to 0.4 MPa , $0.03 \mathrm{~m}^{3}$. Assuming that the pressure and volume are related by $\mathrm{PV}^{\mathrm{n}}=$ constant, find the work interaction duting the process. Is it a work producing system or work absorbing system?
(08 Marks)
3 a. State and explain the first law of thermodynamics for a closed system undergoing a cycle. What is PMM1?
(07 Marks)
b. During one cycle, the working fluid in an engine engages in two work interactions: 15 kJ to the fluid and 44 kJ from the fluid. There are three heat interactions, two of which are known: 75 kJ to the fluid and 40 kJ from the fluid. Evaluate the magnitude and direction of the third heat transfer.
(04 Marks)
c. At the inlet to a certain nozzle, the enthalpy of the fluid passing is $3000 \mathrm{~kJ} / \mathrm{kg}$ and the velocity is $60 \mathrm{~m} / \mathrm{s}$. At the discharge end, the enthalpy is $2762 \frac{\mathrm{~kJ}}{\mathrm{~kg}}$. The nozzle is horizontal and there is negligible heat loss from it.
i) Find the velocity at the exit from the nozzle.
ii) If the inlet area is $0.1 \mathrm{~m}^{2}$ and specific volume at inlet is $0.187 \mathrm{~m}^{3} / \mathrm{kg}$, find the mass flow rate.
iii) If the specific volume at the nozzle exit is $0.498 \mathrm{~m}^{3} / \mathrm{kg}$, find the exit area of the nozzle.

4 a. Establish the equivalence of Kelvin - Planck and Clausius statements of second law of thermodynamics.
(08 Marks)
b. A source ' X ' can supply energy at a rate of $11000 \mathrm{~kJ} /$ minute at $320^{\circ} \mathrm{C}$. A second source ' Y ' can supply energy at a rate of $110000 \mathrm{~kJ} /$ minute at $60^{\circ} \mathrm{C}$. Which source ' X ' or ' Y ' would you choose, to supply energy to an ideal reversible engine, that is to produce a large amount of power, if the temperature of surroundings is $4^{\circ} \mathrm{C}$ ?
(08 Marks)
c. What do you understand by a reversible and an irreversible process? What are the causes of irreversibility of a process?
(04 Marks)

## PART - B

5 a. Define entropy. Show that entropy is a property of the system.
(08 Marks)
b. Define the terms, available and unavailable energy.
(02 Marks)
c. One kg of ice at $-5^{\circ} \mathrm{C}$ is exposed to the atmosphere, which is at $20^{\circ} \mathrm{C}$. The ice melts and comes into thermal equilibrium with the atmosphere. Determine the entropy increase of the universe. Take Cp of ice $=2.093 \frac{\mathrm{~kJ}}{\mathrm{kgK}}$ and latent of fusion of ice $-334 \mathrm{~kJ} / \mathrm{kg}$.
(10 Marks)

6 a. 25 kg of water at $95^{\circ} \mathrm{C}$ is mixed with 35 kg of water at $35^{\circ} \mathrm{C}$, the pressure being taken as constant and temperature of the surroundings being ${15^{\circ}}^{\circ} \mathrm{C}$. Calculate decrease in available energy of the system.
(10 Marks)
b. Define the terms : i) Useful work ; ii) Reversible work iii) Irreversibility. (06 Marks)
c. Define first law efficiency and second law efficiency. How these efficiencies can be improved?
(04 Marks)
7 a. What do you understand by the term degree of superheat" of steam? Show that dryness fraction of a sample of steam measured using combined separating and throttling calorimeter is given by $X=X_{1} X_{2}$, where,
$\mathrm{X}=$ Dryness fraction of steam in steam hain
$\mathrm{X}_{1}=$ Dryness fraction of steam as measured by using separating calorimeter.
$\mathrm{X}_{2}=$ Dryness fraction of steam as measured using throttling calorimeter.
(10 Marks)
b. A pressure cooker contains 1.5 kg of steam at 5 bar and 0.9 dryness when the gas was switched off. Determme the quantity of heat rejected by the pressure cooker when the pressure in the cooker falis to 1 bar .
(10 Marks)
8 a. Under what conditions the behaviour of a real gas approaches closely that of an ideal gas? Write van der wal's equation for a real gas.
(06 Marks)
b. A gas at a pressure of $1.4 \mathrm{MN} / \mathrm{m}^{2}$ and $360^{\circ} \mathrm{C}$ is expanded adiabatically to a pressure of $100 \frac{\mathrm{kN}}{\mathrm{m}^{2}}$. The gas is then heated at constant volume until it attains $360^{\circ} \mathrm{C}$ when its pressure is found to be $220 \mathrm{kN} / \mathrm{m}^{2}$. Finally it is compressed isothermally to the original pressure of $1.4 \frac{\mathrm{MN}}{\mathrm{m}^{2}}$. Sketch the process on P-V and T-S diagrams. For 0.23 kg of gas, evaluate the following : work transfer, heat transfer, change in internal energy and change in entropy during each process. Assume the following data for the gas:
$\mathrm{C}_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$
$\mathrm{C}_{\mathrm{v}}=0.718 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$
$\mathrm{R}=0.287 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$
$\gamma=1.4$
(14 Marks)


## Third Semester B.E. Degree Examination, May/June 2010 Mechanics of Materials

Time: 3 hrs .
Max. Marks:100

## Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. Explain : i) Poisson's ratio ; ii) Young's modulus.
(04 Marks)
b. Mention the assumptions made in the theory of simple stress and strain and derive the equation $\mathrm{dl}=\mathrm{PL} / \mathrm{AE}$.
(06 Marks)
c. A brass bar having a cross sectional area of $1000 \mathrm{~mm}^{2}$ is subjected to axial forces as shown in Fig.1(c). Determine the total elongation of the bar if $\mathrm{E}=105 \mathrm{GP}$.
(10 Marks)

Fig.1(c)


2 a. Define : i) Modulus of rigidity ; ii) Volumetric strain.
(04 Marks)
b. Explain the reason for development of stress in bars, when their temperature rises or falls. Accordingly calculate the nature and magnitude of stress induced in the rod of 2 m length and 20 mm diameter, when its temperature rises by $70^{\circ} \mathrm{C}$, with both ends constrained. Take $E=1 \times 10^{5} \mathrm{~N} / \mathrm{mm}^{2}$ and $\alpha=1.2 \times 10^{-5} / \rho \mathrm{C}$.
(06 Marks)
c. A composite section comprises of a steel fube 10 cm internal diameter and 12 cm external diameter, fitted inside a brass tube of 14 cm internal diameter and 16 cm external diameter. The assembly is subjected to a compressive load of 500 kN . Find the load carried by the tube and the stresses induced in them. The length of tube is 150 cm . Take $\mathrm{E}_{\text {stel }}=200 \mathrm{GPa}$ and $\mathrm{E}_{\text {brass }}=100 \mathrm{GPa}$. What is the change in length of tubes?
(10 Marks)
3 a. The longitudinal strain of a cylindrical bar of diameter 3 cm and length 1.5 m is four times the lateral strain during a tensile test. Determine the modulus of elasticity and bulk modulus. Also determine the change in volume when the bar is subjected to a hydrostatic pressure of 100 MPa . Take $=100 \mathrm{GPa}$.
(10 Marks)
b. The state of stress in a two dimensionally stressed body is shown in Fig.3(b). Determine the principal planes, principal stress, maximum shear stress and their planes. Schematically represent these planes on $\mathrm{x}-\mathrm{y}$ coordinates.
(10 Marks)

Fig.3(b)


1 of 2

4 a. Obtain an expression for the volumetric strain of a thin cylinder, subjected to internal fluid pressure.
(08 Marks)
b. Determine the hoop stress and radial pressure across the section of a thick cylinder of internal diameter 40 cm and thickness 10 cm , when it contains a fluid at a pressure of $8 \mathrm{~N} / \mathrm{mm}^{2}$. Also sketch the distribution of hoop stress and radial pressure.
(12 Marks)

## PART - B

5 a. Explain the terms:
i) Sagging bending moment
ii) Hogging bending moment
iii) Point of contra flexure.
(06 Marks)
b. Draw shear force and bending moment diagrams for the loading pattern on the beam shown in Fig.5(b). Indicate where the inflexion and contra flexure points are located. Also locate the maximum bending with its magnitude.
(14 Marks)

a. Show that the maximum shear stress for a reetangular section is 1.5 times the average shear stress.
(06 Marks)
b. A 'T' section of flange $120 \mathrm{~mm} \times 12 \mathrm{~mm}$ and overall depth 200 mm , with 12 mm web thickness is loaded, such that, at a section it has a moment of 20 kNM and shear force of 120 kN . Sketch the bending and shear stress distribution diagram, marking the salient values.
(14 Marks)
7 a. Distinguish betweê slope and deflection. Explain the same with examples for a simply supported beam and a cantilever.
(06 Marks)
b. A beam AB of 6 m span simply supported at the ends and is loaded with a point load of 6 kN at 2 m from left support and uniformly distributed load of $2 \mathrm{kN} / \mathrm{m}$ for the second half of the beam. Find:
i) Denlection at mid span
ii) Maximum deflection
iii) Slope at left support

Take $\mathrm{E}=20 \mathrm{GPa}$ and $\mathrm{I}=2 \times 10^{7} \mathrm{~mm}^{4}$.
(14 Marks)
8 a. Derive the expression for Euler's buckling load for a column with its one end fixed and the other end free.
(06 Marks)
b. A solid shaft transmits 250 kW at 100 rpm . If the shear stress is not to exceed 75 MPa , what should be the diameter of the shaft? It this shaft is to replaced by a hollow shaft, whose diameter ratio is 0.6 , determine the size and percentage having in weight, the maximum shear stress being the same.
(14 Marks)

|  |  |  |  |  |  |  |  |
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# Third Semester B.E. Degree Examination, May/June 2010 <br> Manufacturing Process - I 

Time: 3 hrs .

## Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

## PART - A

1 a. Explain the steps involved in making a sound casting.
(08 Marks)
b. Explain the different allowances on the pattern.
(06 Marks)
c. Sketch and explain a match plate pattern.
(06 Marks)
2 a. What are the required properties of moulding sand?
(06 Marks)
b. Discuss briefly how castings are cleaned.
(06 Marks)
c. Sketch and explain a squeezer type of molding machine
(08 Marks)

3 a. Sketch and explain a centrifugal casting machine highlighting its applications. (12 Marks)
b. Explain the steps involved in shell molding
(08 Marks)

4 Sketch and explain the following, highlighting its field of application :
i) Cupola
ii) Pit furnace.
(20 Marks)
PART - B
5 a. Sketch and explain a TIG welding set up and its uses.
(10 Marks)
b. Explain the different element involved in a gas welding set up.
(10 Marks)

6 a. Explain the principle of resistance welding. How does a spot welding set up work?( 10 Marks)
b. Sketch and explain the Thermit welding set up with its field of application.
(10 Marks)

7 a. Explain how shrinkage in welds can be minimized. How residual stresses in welds can be removed?
b. Explain the different defects present in welded structures.
(08 Marks)

8 a. What is brazing? How is it carried out?
(08 Marks)
b. Explain the following inspection methods, with relevant sketches :
i) Magnetic particle inspection
ii) Radiographic method.
(12 Marks)


MATDIP301

Third Semester B.E. Degree Examination, May/June 2010 Advanced Mathematics - I
Time: 3 hrs .
Max. Marks:100

## Note: Answer any FIVE full questions.

1 a. Express the complex number $\frac{(1+\mathrm{i})(1+3 \mathrm{i})}{1+5 \mathrm{i}}$ in the form $\mathrm{x}+\mathrm{iy}$.
(06 Marks)
b. Prove that $(1+i)^{n}+(1-i)^{n}=2^{\frac{n}{2}+1} \cos \left(\frac{n \pi}{4}\right)$.
c. Expand $\cos ^{8} \theta$ in a series of cosines multiples of $\theta$.
(07 Marks)
2 a. Find the $\mathrm{n}^{\text {th }}$ derivative of $\mathrm{e}^{\mathrm{ax}} \sin (\mathrm{bx}+\mathrm{c})$.
(06 Marks)
b. If $y=a \cos (\log x)+b \sin (\log x)$, prove that $x^{2} y_{n+2}+(2 n+1) x y_{n+1}+\left(n^{2}+1\right) y_{n}=0$.
(07 Marks)
c. Find the $n^{\text {th }}$ derivative of $\frac{x}{(x-1)(2 x-3)}$
(07 Marks)

3 a. State Taylor's theorem and expand the polynomial $2 x^{3}+7 x^{2}+x-6$ in powers of $(x-1)$.
(06 Marks)
b. Expand $\tan \mathrm{x}$ in ascending powers of x using MacLaurin's theorem upto the term containing $\mathrm{x}^{4}$.
(07 Marks)
c. If $Z=\frac{x^{2}+y^{2}}{x+y}$ prove that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^{2}=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$.
(07 Marks)

4 a. If $u=\tan ^{-1}\left(\frac{x^{3}+y^{3}}{x-y}\right)$, prove that $x \frac{\partial u}{\partial x}+y \frac{\partial u}{\partial y}=\sin 2 u$.
(06 Marks)
b. If $u=f(x, y)$ where $x=r \cos \theta$ and $y=r \sin \theta$, prove that $\left(\frac{\partial u}{\partial x}\right)^{2}+\left(\frac{\partial u}{\partial y}\right)^{2}=\left(\frac{\partial u}{\partial r}\right)^{2}+\frac{1}{r^{2}}\left(\frac{\partial u}{\partial \theta}\right)^{2}$.
(07 Marks)
c. If $u=x^{2}-2 y, v=x+y+z$ and $w=x-2 y+3 z$, find the value of $J\left(\frac{u, v, w}{x, y, z}\right)$.
(07 Marks)

5 a. Obtain the reduction formula for $\int \sin ^{m} x \cos ^{n} x d x$.
(06 Marks)
b. Evaluate $\int_{0}^{a} \frac{x^{7}}{\sqrt{a^{2}-x^{2}}} d x$.
(07 Marks)
c. Evaluate $\int_{0}^{1} \int_{0}^{x} e^{\left(\frac{y}{x}\right)} d y d x$.
(07 Marks)

6 a. Evaluate $\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \sqrt{0} \sqrt{1-x^{2}-y^{2}} x y z d z d y d x$.
(06 Marks)
b. Prove that $\beta(m, n)=\frac{\sqrt{m} \mid \bar{n}}{\mid \overline{m+n}}$.
(07 Marks)
c. Show that $\int_{0}^{\pi / 2} \sqrt{\sin \theta} d \theta \times \int_{0}^{\pi / 2} \frac{d \theta}{\sqrt{\sin \theta}}=\pi$.
(07 Marks)

7 a. Solve $3 \mathrm{e}^{\mathrm{x}} \tan \mathrm{y} d \mathrm{dx}+\left(1-\mathrm{e}^{\mathrm{x}}\right) \sec ^{2} \mathrm{y} \mathrm{dy}=0$.
(06 Marks)
b. Solve $x^{2} y d x=\left(x^{3}+y^{3}\right) d y$.
c. Solve $x \frac{d y}{d x}+y=x^{3} y^{6}$.

8 a. Solve $\frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}=x^{2}+2 x+4$.
(06 Marks)
b. Solve $\frac{d^{2} y}{d x^{2}}+3 \frac{d y}{d x}+2 y=4 \cos ^{2} x$.
(07 Marks)
c. Solve $\frac{d^{3} y}{d x^{3}}+2 \frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}=e^{-x}+\sin 2 x$.

